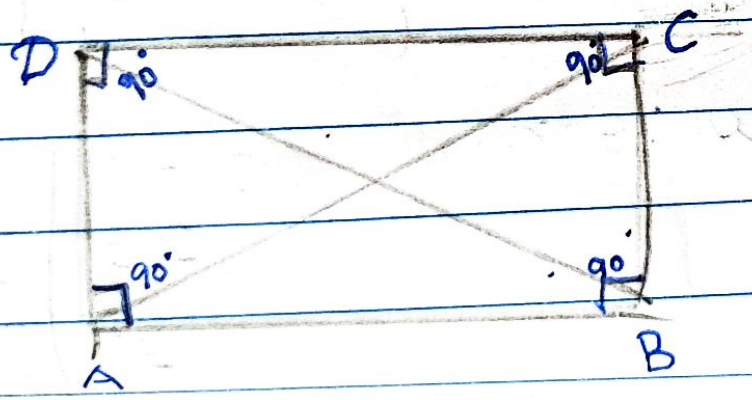
4. Quadrilaterals

Quadrilaterals:

A quadrilateral is a polygon with four sides.

Example: Square, Rectangle, Parallelogram, Rhombus, **trapezium**, kite.

Rectangles:



Properties of Rectangle.

① All the angles of a rectangle are 90°

$\angle A = \angle B \quad \angle C = \angle D = 90^\circ$

② Opposite sides of a rectangle are equal.

$AB = CD$

$DA = CB$

③ The opposite sides of a rectangle are parallel to each other

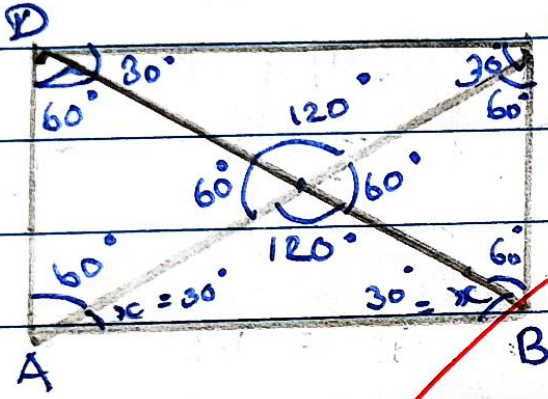
$AB \parallel DC \quad DA \parallel BC$

④ The diagonal of a rectangle are of equal length and they bisect each other.

$AC = DB$

$OA = OB = OC = OD$.

Angles between diagonals



$\angle BOC = \angle AOD = 60^\circ$

$\angle DOC = \angle AOB = 120^\circ$

$\angle O = 60^\circ + 60^\circ + 120^\circ + 120^\circ = 360^\circ$

$120^\circ + x + x = 180^\circ$ (Angle sum property)

$2x + 120^\circ = 180^\circ$

$2x = 180^\circ - 120^\circ$

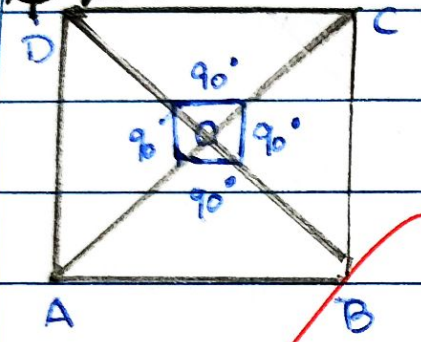
$2x = 60^\circ$

$x = \frac{60^{30}}{2}$

$x = 30^\circ$

$\angle A = \angle B = \angle C = \angle D = 30^\circ + 60^\circ + = 60^\circ$

Square :-



① The angles of a square are 90°

$\angle A = \angle B = \angle C = \angle D = 90^\circ$

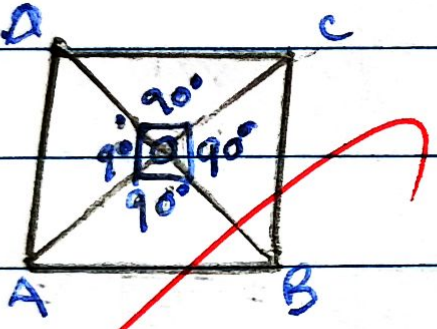
② All the sides of a square are equal to each other.

$AB = BC = CD = DA$.

③ The opposite sides of a square are parallel to each other.

AB // CD AD // BC

④. The diagonals of square are of equal and they bisect each other at 90°



⑤. The diagonal of square divides the angle of a square into equal halves.

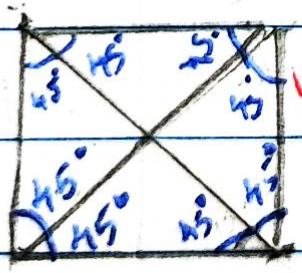
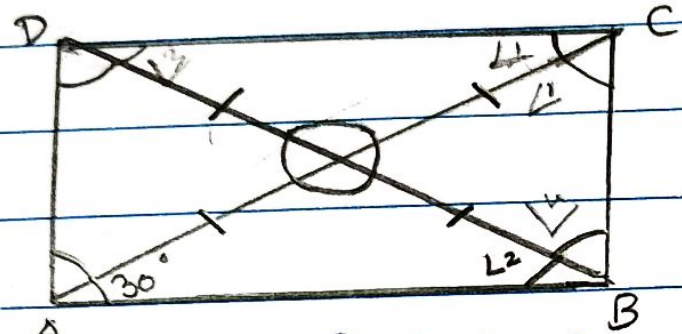


Figure it out : (page no : 94)

(1) i)



$\angle 1 = 30^\circ$ [Angle opposite to equal sides]

$\angle 2 = 30^\circ$, [Alternate interior angle are equal].

$\angle 3 = 30^\circ$

$30^\circ + 30^\circ + \angle 4 = 180^\circ$ [Angle sum property].

$60^\circ + \angle 4 = 180^\circ$

$\angle 4 = 180^\circ - 60^\circ$

$= 120^\circ$

$\angle 5 = 120$ [vertically opposite angles]

$\angle 6 + 30^\circ = 90^\circ$

$\angle 6 = 90^\circ - 30^\circ$

$\angle 6 = 60^\circ$ [each angle of Rectangle is 90°]

$\angle 7 + 30^\circ = 90^\circ$ [each angle of rectangle

$\angle 7 = 90^\circ - 30^\circ$ is 90°]

$\angle 7 = 60^\circ$

$\angle 8 = 60^\circ$ Alternate angles

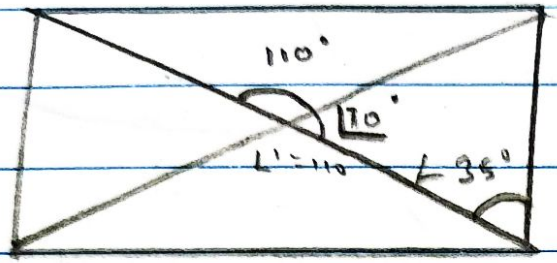
$\angle 9 = 60^\circ$

$\angle 10 + 60^\circ + 60^\circ = 180^\circ$ [Angle sum

$\angle 10 = 180^\circ - 120^\circ = 60^\circ$ property].

$\angle 11 = 60^\circ$ [vertically opposite angle are equal]

(i)(ii)



$\angle 1 = 110^\circ$ (vertically opposite angles are equal)

$\angle 2 + 110^\circ = 180^\circ$ (linear pair)

$\angle 2 = 180^\circ - 110^\circ$

$\angle 2 = 70^\circ$

$\angle 3 = 70^\circ$ (vertically opposite angles are equal).

let $\angle 4 = \angle 5 = x$

$x + x + 110^\circ = 180^\circ$

$2x + 110^\circ = 180^\circ$

$2x = 180^\circ - 110^\circ$

$2x = 70^\circ$

$x = \frac{70}{2}$

$x = 35^\circ$

$\angle 4 = 35^\circ$

$\angle 5 = 35^\circ$.

$\angle 6 = 35^\circ$ (Alternate interior angles are equal)

$\angle 7 = 35^\circ$ (Alternate interior angles are equal)

Each angle of a rectangle $= 90^\circ$

$\angle 8 + 35^\circ = 90^\circ$

$\angle 8 = 90^\circ - 35^\circ$

$\angle 8 = 55^\circ$

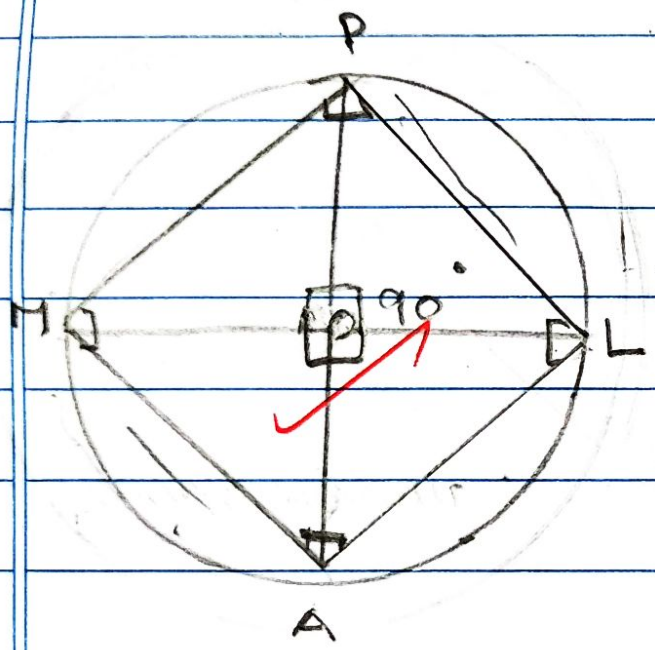
$\angle 9 = 90^\circ - 35 = 55^\circ$

Alternate interior angles are equal

$\angle 10 = 55^\circ$

$\angle 11 = 55^\circ$.

(2).



Ans: APML is a square.

$\Rightarrow PL \perp AM$

$\Rightarrow$ Diagonals are equal and bisect each other at 90°

$\Rightarrow$ Each angles are 90° $\angle A = \angle K = \angle P$

$\angle M = 90°$

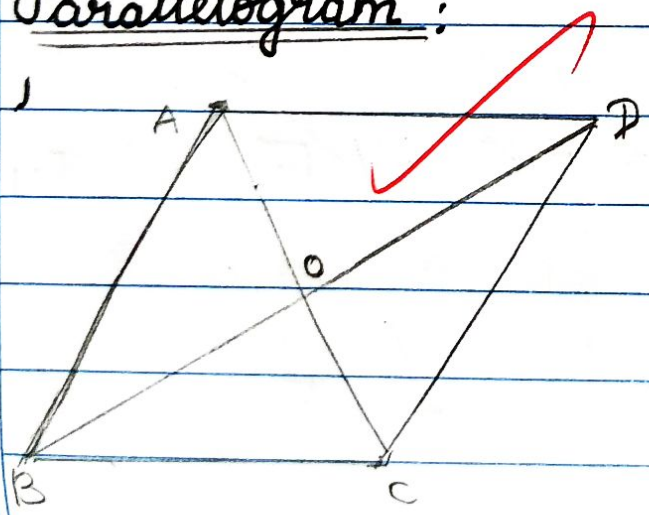
$\Rightarrow \therefore$ APML is a square.

⑤. Ans: * No, this cannot be the definition of the Rectangle.

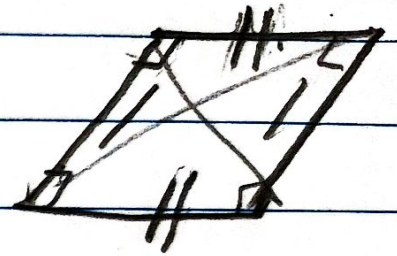
* A quadrilateral with opposite sides are parallel and equal is a parallelogram.

* But all parallelograms are not rectangles. Rectangle means all angles to be right angles.

Parallelogram:



Properties of parallelogram:

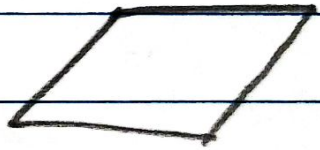


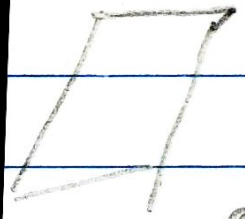
① ⇒ The opposites sides are equal.

$\overline{AB} = \overline{CD}$ $\overline{AD} = \overline{BC}$.

② ⇒ The opposite sides of a parallelogram are parallel

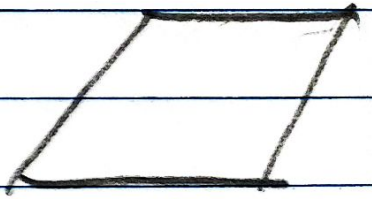
$AD \parallel BC$ $AB \parallel DC$





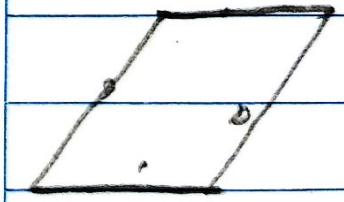
③ ⇒ The opposite angles of parallelogram is equal.

$\angle A = \angle C$, $\angle B = \angle D$



④ ⇒ The sum of the adjacent angles of a parallelogram is 180°

$\angle B + \angle C = 180^\circ$ $\angle A + \angle D = 180^\circ$



(3) - $\angle A + \angle B = 180^\circ$ $\angle D + \angle C = 180^\circ$

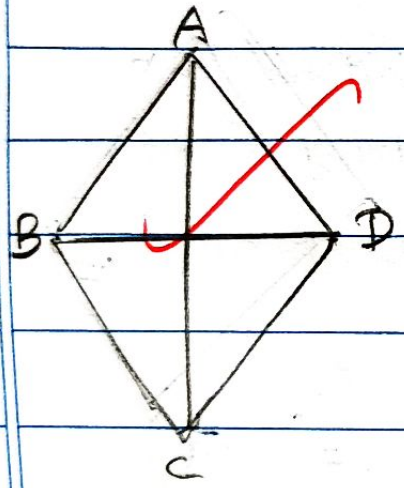
(4) ⇒ The diagonals of a parallelogram bisect each other.

$OB = OD$, $OA = OC$.

In Parallelogram diagonals are not equal

Rectangle satisfy all the properties of parallelogram

Rhombus:



Properties of Rhombus:

⇒ All the sides of a Rhombus are

equal to each other.

$AB = BC = CD = DA$

⇒ The opposite sides of a rhombus are parallel to each other.

$\underline{AB} \parallel DC$, $AD \parallel BC$

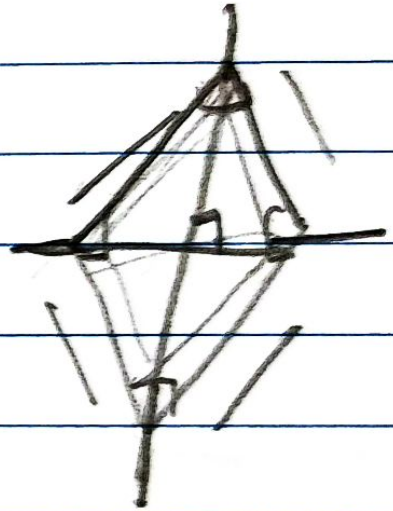
⇒ The opposite angles of a rhombus are equal

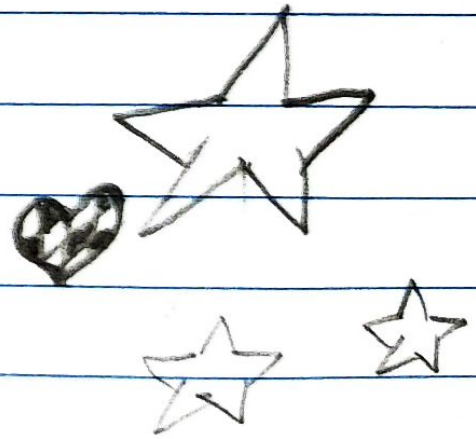
$\angle A = \angle C$ $\angle B = \angle D$.

⇒ The sum of adjacent angles of a rhombus is 180°

$\angle A + \angle B = 180^\circ$ $\angle B + \angle C = 180^\circ$ $\angle C + \angle P = 180^\circ$ $\angle D + \angle A = 180^\circ$

Square satisfy all the properties of rhombus.





⇒ The diagonals of rhombus bisect each other at 90°

OA = OC , OB = OD.

⇒ The diagonals of rhombus bisect its angles.

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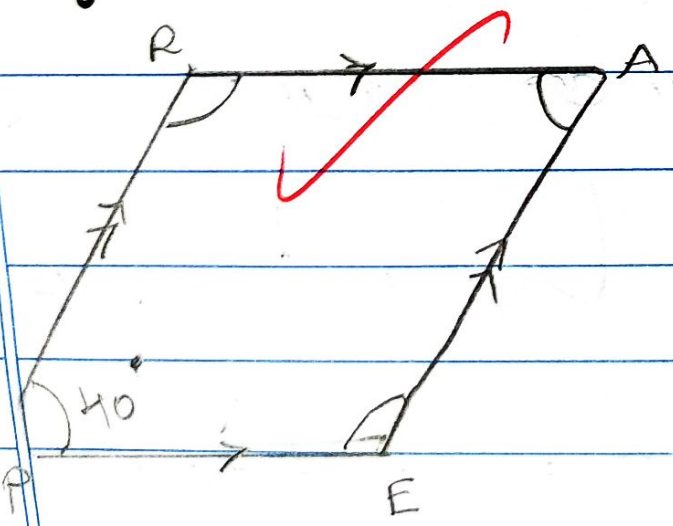
① i) Figure it out: (page no : 102)



$\angle P + \angle E = 180°$ (sum of the adjacent angles are equal).

$40° + \angle E = 180°$

$\angle E = 180° - 40°$

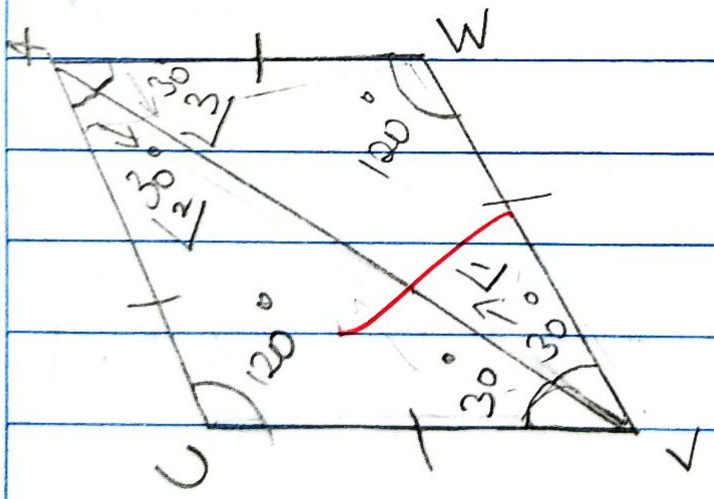
$= 140°$

$E = 140°$

$\angle E = \angle R = 140°$ (opposite angles are equal)

$\angle P = \angle Q = 40°$ (opposite angles are equal).

iii)



$\angle 1 = 30^\circ$ (diagonals of Rhombus bisect each other)

$\angle 2 = 30^\circ$ (Opposite interior angles are equal)

$\angle 3 = 30^\circ$

$U + V = 180^\circ$ (Sum of the diagonals are 180°)

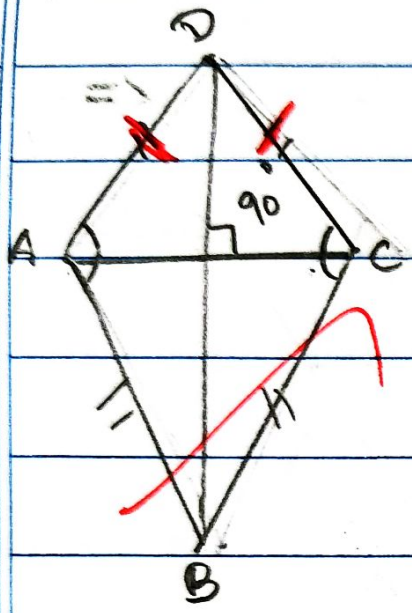
$U + 60 = 180^\circ$

$U = 180^\circ - 60^\circ$

$U = 120^\circ$

$U = \angle W = 120^\circ$ (opposite angles are equal)

Kite :



Properties of kite :

⇒ Adjacent sides of kite are equal

$AB = BC$, $AD = DC$.

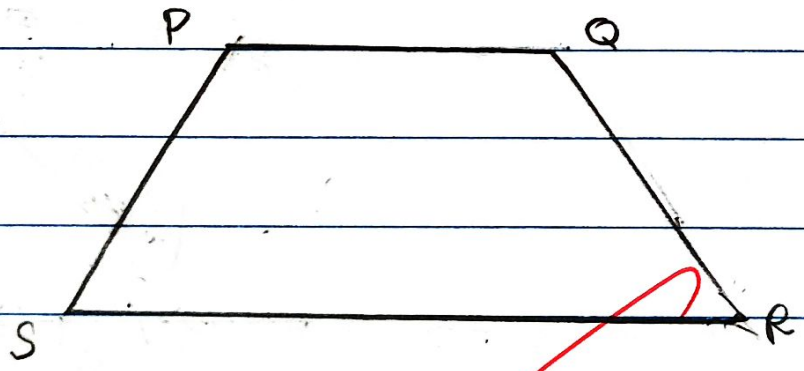
⇒ One diagonal of a kite bisect at 90°.

⇒ One pair of opposite angle is equal $\angle A = \angle C$.

Trapezium :

A trapezium is a quadrilateral with atleast one pair of parallel sides is :

Isosceles Trapezium :

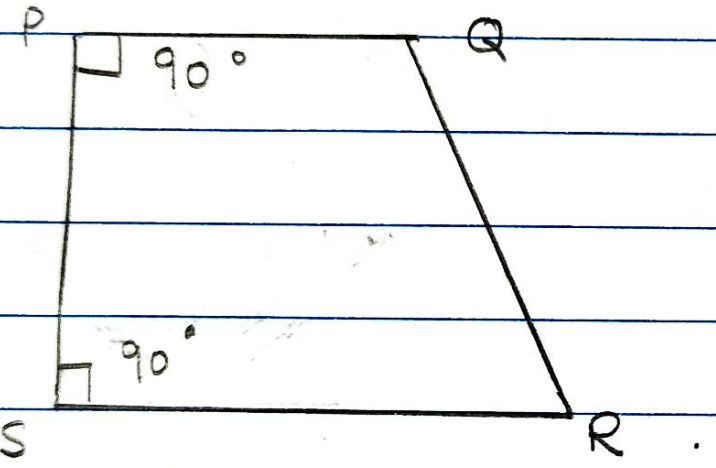


⇒ A trapezium in which the non-parallel sides are equal is called an isosceles trapezium.

$PS = QR$

$\angle P + \angle S = 180^\circ$, $\angle Q + \angle R = 180^\circ$.

Right trapezium :



A trapezium that has two right angles are called right trapezium

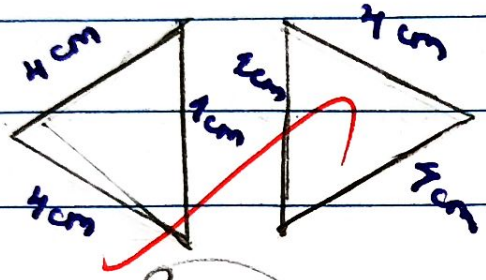
$\angle P = \angle S = 90^\circ$.

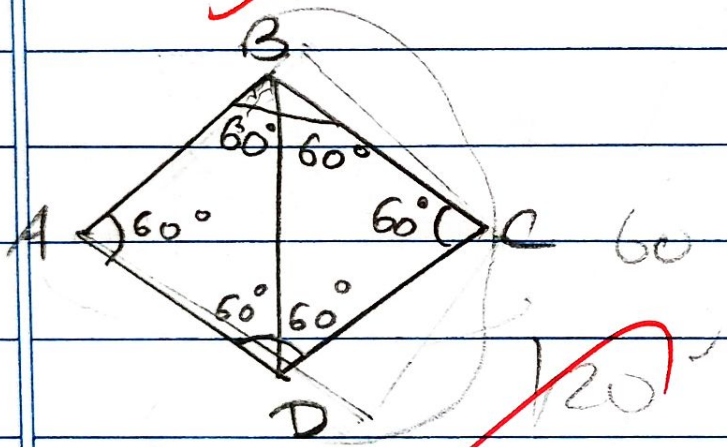
⇒ In an isosceles trapezium, the angles opposite to the equal sides are

equal.

Figure it out (page no: 107)

① .





Sides:

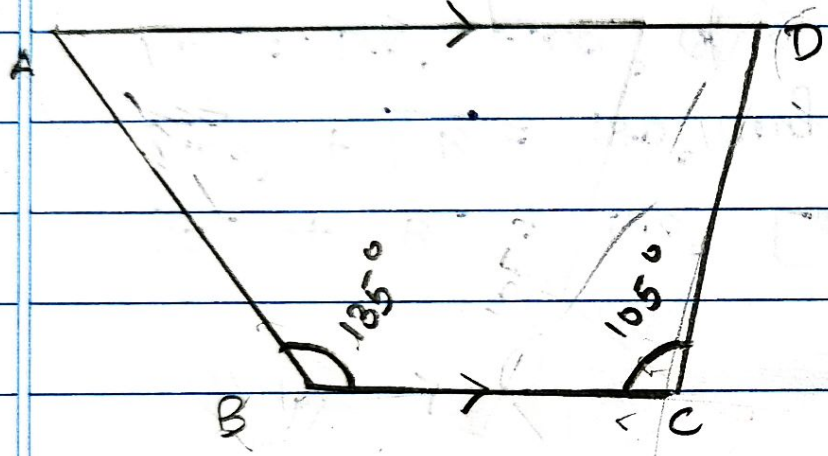
$AB = BC = CD = DA = 4$ cm

Angles:

$\angle A = \angle C = 60°$

$\angle B = \angle D = 120^\circ$ [each angle of an equilateral triangle is equal].

(2) i)



$\angle A + \angle B = 180^\circ$

$\angle A + 135^\circ = 180^\circ$ [sum of the

$180^\circ - 135^\circ = \angle A$ interior angles on the

$\angle A = 45^\circ$ same sides of transversal

$= 180^\circ$].

$\angle C + \angle D = 180^\circ$

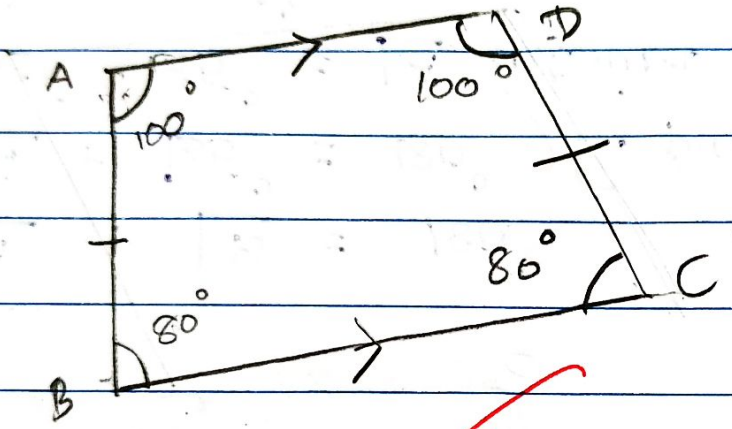
$105^\circ + \angle D = 180^\circ$

$180^\circ - 105^\circ = \angle D$

$\angle D = 75^\circ$

$$\begin{array}{r} 17 \\ 18\not{0}^{10} \\ 105 \\ \hline 075 \end{array}$$

ii)



$\angle A + \angle B = 180^\circ$ [sum of the interior

$100^\circ + \angle B = 180^\circ$ angles on the same side

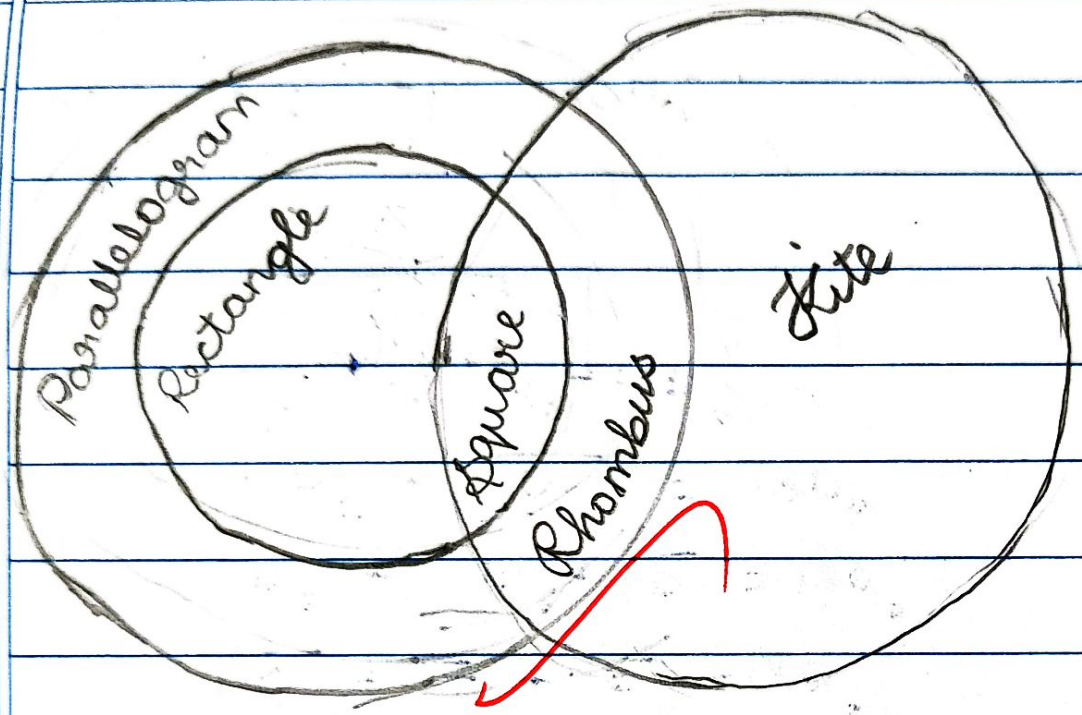
$\angle B = 180^\circ - 100^\circ$ of a transversal $= 180^\circ$]

$\angle B = 80^\circ$

$\angle D = 100^\circ$

$\angle C = 80^\circ$ [In a isosceles triangle the angles opposite to the equal sides are equal].

④

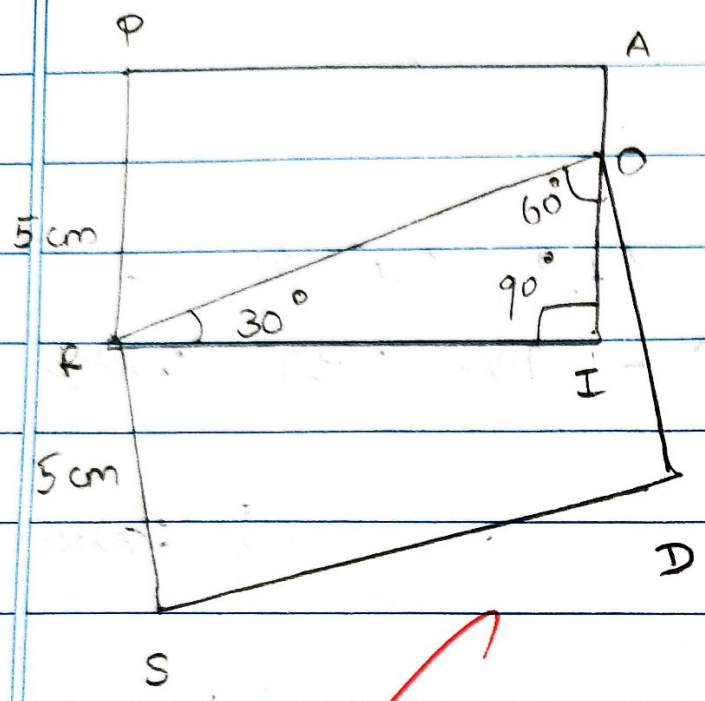


i) Ans: Square and rhombus

ii) Ans: Square.

iii) Ans: No. All the Rhombus are kite but all the kites are not rhombus.

(5)



In △ RIO,

$\angle R + \angle I + \angle O = 180^\circ$

$30^\circ + 90^\circ + \angle O = 180^\circ$

$120^\circ + \angle O = 180^\circ$

$\angle O = 180^\circ - 120^\circ$

$\boxed{\angle O = 60^\circ}$

In rectangle ROOS,

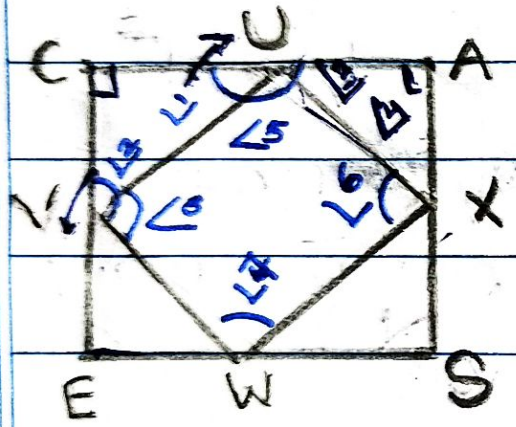
$60^\circ + \angle 1 = 90^\circ$

$\angle 1 = 90^\circ - 60^\circ$

$\angle 1 = 30^\circ$

$\therefore \angle IOD = 30^\circ$.

⑦.



Side :

In $\triangle UCV$ and $\triangle UAX$

$CU = UA$ (S)

$\angle C = \angle A$ (A)

$CV = AX$ (S)

SAS,

$\triangle UCV = \triangle UAX$

$UV = UX \rightarrow (1)$

Similarly,

$VU = VW \rightarrow (2)$

$VW = WX \rightarrow (3)$

$WX = XU \rightarrow (4)$

From (1), (2), (3), (4)

$UV = VW = WX = XU$

∴ All the four sides of the quadrilateral are equal → (I).

Angles:

In $\triangle VCV$

$CU = CV$

$\angle 1 = \angle 2$

$\angle 1 + \angle C + \angle 2 = 180^\circ$

$\angle 1 + 90^\circ + \angle 1 = 180^\circ$

$2\angle 1 + 90^\circ = 180^\circ$

$2\angle 1 = 180^\circ - 90^\circ$

$2\angle 1 = 90^\circ$

$\angle 1 = \frac{90}{2} = 45^\circ$

$\angle 1 = \angle 2 = 45^\circ$

$\angle 3 = \angle 4 = 45^\circ$

$\angle 1 + \angle 5 + \angle 3 = 180^\circ$

$45^\circ + \angle 5 + 45^\circ = 180^\circ$

$90^\circ + \angle 5 = 180^\circ$

$\angle 5 = 180^\circ - 90^\circ$

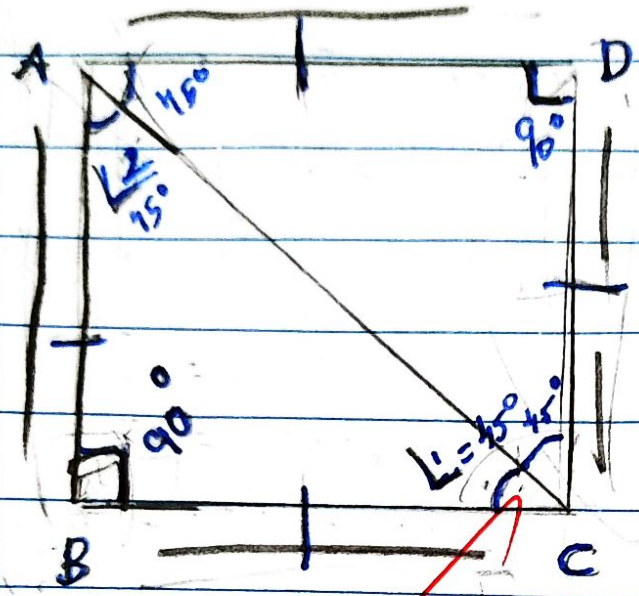
$\angle 5 = 90^\circ$

Similarly,

$\angle 6 = \angle 7 = \angle 8 = 90^\circ$ Each angles are $90^\circ \Rightarrow$ (II)

From I, II UXVW is a square.

8.



Δ ABC and Δ ADC

AB = AD (side)

BC = CD (side)

AC = AC (Common side)

$\Delta ABC \cong \Delta ADC$

$\angle B = \angle D = 90°$

ΔABC

$\angle 1 + \angle 2 + 90° = 180°$

$\angle 1 + \angle 1 + 90° = 180°$

$2\angle 1 + 90° = 180°$

$2\angle 1 = 180° - 90°$

$2\angle 1 = 90°$

$= 90 45°$

$\angle A = 45° + 45° = 90°$

$\angle C = 45° + 45° = 90°$

$\angle A = \angle B = \angle C = \angle D = 90°$

All four sides equal

All four angles $90°$

So, It is a square

In ΔABC

$\angle 1 + \angle 2$

$\angle 1 + \angle 2 + 90^\circ = 180^\circ$

$\angle 1 + \angle 1 + 90^\circ = 180^\circ$

$2\angle 1 + 90^\circ = 180^\circ$

$2\angle 1 = 180^\circ - 90^\circ$

$2\angle 1 = 90$

$\angle 1 = \frac{90}{2}^{45}$

$\angle 1 = \angle 2 = 45^\circ$

$\angle A = 45^\circ + 45^\circ = 90^\circ$

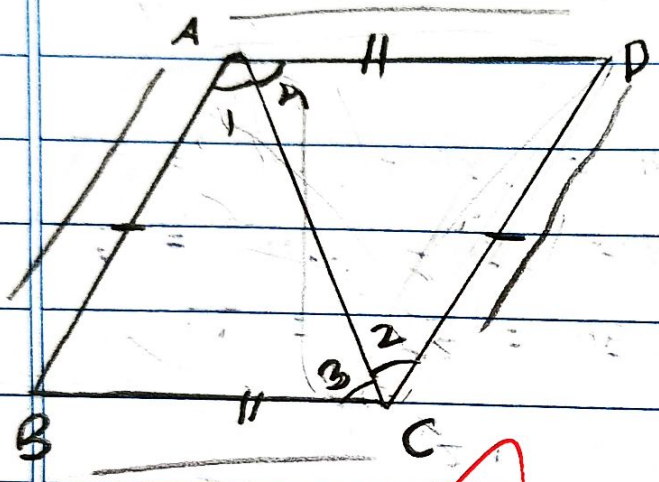
$\angle C = 45^\circ + 45^\circ = 90^\circ$

$\angle A = \angle B = \angle C = \angle D = 90^\circ$

* All the four sides are equal
* All the angles are 90° (four)

* Yes, we will get a square.

9.



$\triangle ABC$ and $\triangle ADC$

$AB = DC$ (SS)

$BC = AD$ (SS)

$AC = AC$ (common side)

$\therefore \triangle ABC \cong \triangle ADC$

$\angle 1 = \angle 2$

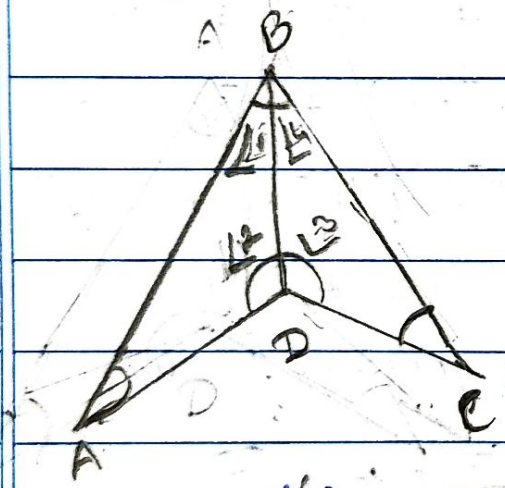
$\angle 3 = \angle 4$

Also, there are alternate interior angles formed by sides AB and CD and AD, BC and transversal AC.

$\therefore$ AB $\parallel$ CD and AD $\parallel$ BC

$\Rightarrow$ ABCD is a parallelogram.

10.



* Join BD
* Now ABD and BDC are two triangles

In $\triangle ABD$

$\angle A + \angle 1 + \angle 2 = 180^\circ \rightarrow$ ① (Angle sum property).

In $\triangle BDC$

$\angle C + \angle 3 + \angle 4 = 180^\circ \rightarrow$ ② (Angle sum property)

① + ②

$\angle A + \angle 1 + \angle 2 + \angle C + \angle 3 + \angle 4 = 180^\circ + 180^\circ$

$\angle A + (\angle 1 + \angle 2) + (\angle 3 + \angle 4) + \angle C = 360^\circ$

$\angle A + \angle D + \angle B + \angle C = 360^\circ$

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

Ans: Yes.